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Graphic Conversion of Stellar Co-ordinates.

By the Rev. A. Freeman, M.A.

When a direct-vision altazimuth or a Russian diagonal instrument is to be used for observation of an object off the meridian, the object must first be found by reference to a star-map and by personal comparison of this map with the constellation in which the object appears. A celestial globe, provided with a brass meridian and a moveable vertical circle, would save time by giving at once an approximation to the altitude and azimuth of the object at the time of observation. But a celestial globe, large enough for the purpose, is both expensive and cumbrous. All that is required is a carefully constructed diagram upon which shall be drawn the hour-circles and polar-distance circles of the visible celestial hemisphere, as they are situated with reference to vertical circles and circles of equal altitude at the latitude of the observatory.

The *Nautical Almanac* gives for each day the time of transit of the first point of *Aries*, add to this the right ascension of the object, as given by its ephemeris or by a star-catalogue, and subtract the time of observation, and the result is the hour-angle of the object. Its polar-distance is also known. Hence the place of the object on the diagram is defined, and inspection will show how it is situated as regards altitude and azimuth. Thus the observing instrument may at once be set upon it.

A diagram of this kind has been drawn to exhibit the necessary circles, as they are stereographically projected from the nadir, upon the tangent-plane at the zenith of the celestial sphere in latitude 52½°. The horizontal circle is twenty-five inches in diameter, and successive circles of the same system differ by 5°. The latitude is that of central England. Birmingham (lat. 52° 29′) is the largest town which the diagram serves.* It was afterwards discovered to be equally suitable for Berlin (lat. 52° 31′). This intermediate latitude contributes to the clearness of the drawing, and has saved about half the labour required for the calculation of the sites of the polar-distance circles.

The stereographic projection has been adopted because it exhibits the whole visible hemisphere, and because all the lines required to be drawn are either circles or straight lines. This is the result of a well-known property of this projection. The angle of intersection of any two curves on the sphere is unchanged by projection. Circles are projected as circles, and all small figures are similar to their projections. Hence the four-sided figure between any two successive meridians and any two successive polar-distance circles, is projected into a four-sided figure bounded by circles, which intersect at right-angles. On this

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* Norwich, Bury St. Edmunds, Peterborough, Cambridge, Leicester, Rugby, Shrewsbury, and Worcester, are each within about 15′ of this latitude.

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account it is easy to measure with accuracy the co-ordinates of a star-position falling within the figure.

The projections of the meridians, passing through two fixed points, the north and south poles, are circles having a common radical axis. The projections of the same-polar-distance circles, being circles orthogonal to the meridians, are also circles with a common radical axis, viz. the line of centres of meridians. This is easily proved by geometry.

For, if N and S (fig. 1) be the north and south poles, C and C′ the centres of two meridian-circles passing through them, any point P on the radical axis, N S, is such, that the tangents drawn to the two meridians are equal, and a circle with centre P and radius equal to either tangent, will cut each meridian at right angles. Also, if P′ be the centre of a second orthogonal circle, the tangents drawn from C to the two orthogonal circles are each equal to the radius of the circle C; and the tangents drawn from C′ are each equal to the radius of the circle C′. Hence, C C′ is the radical axis of the orthogonals.

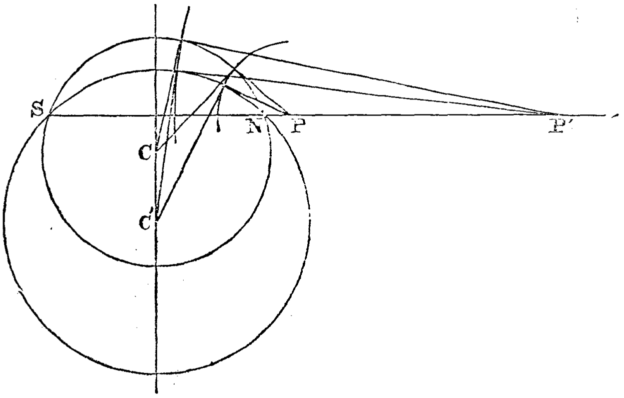

Fig. 1.

The next step is to discover mathematical expressions for the radii and centre co-ordinates of all the circles required to be drawn.

(1.) *Circles of same-zenith-distance.*—The nadir being the point of projection, and the tangent-plane at the zenith being the plane of projection, if z, measured along a vertical circle, is the zenith-distance of any point in a same-zenith-distance circle, the radius of the projected circle is $2\,r\tan\frac{z}{2}$; and all these circles are concentric with the zenith.

(2.) *Vertical circles,* or *same-azimuth circles.*—These are evidently projected into straight lines, radiating from the zenith, and the angle between a pair of successive straight lines is in the diagram 5°.

(3.) *Same-polar-distance circles.*—These circles cut the meridian at right-angles, and are bisected by it. Now, the meridian is projected into a straight line. Hence, a same-polar-distance

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circle is projected into a circle cut orthogonally by the meridian, having, therefore, its centre on the meridian.

Let c be the co-latitude, and Δ the common polar-distance, then the ends of the diameter of the projected circle are at distances from the zenith equal to

$$2\,r\tan\frac{1}{2}(c+\Delta) \qquad \text{and} \qquad 2\,r\tan\frac{1}{2}(c-\Delta)$$

Call these d_2 and d_1, respectively, then

$$\frac{1}{2}(d_2 - d_1) = \text{radius of the polar distance circle,}$$

$$\frac{1}{2}(d_2 + d_1) = \text{distance of its centre from the zenith.}$$

By making $\Delta = 0^\circ$, or 180° successively, we find that the projections of the north and south poles are at distances

$$2\,r\tan\frac{c}{2} \quad \text{and} \quad 2\,r\cot\frac{c}{2}$$

on each side of the zenith. The distance between the poles is consequently $4\,r$ cosec c. Hence,

$$(2\,r\ \text{cosec.}\ c - 2\,r\tan\frac{1}{2}c) = 2\,r\cot c$$

is the distance from the zenith of the line of centres of meridians, or hour-circles passing through the poles.

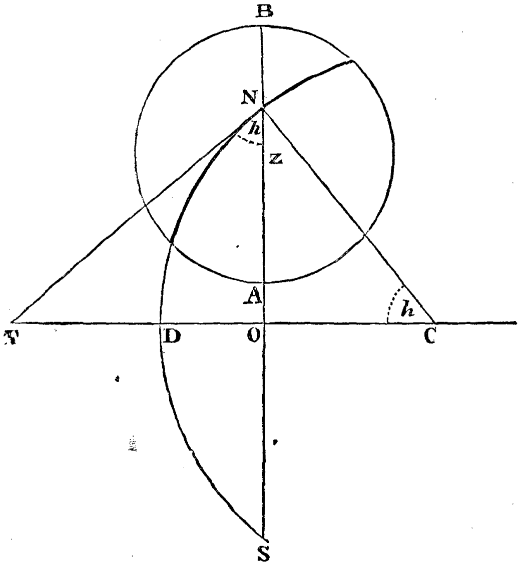

Fig. 2.

(4.) *Same-hour-angle circles,* or *meridians.*—Let Z, N, S, (fig. 2) be respectively the projections of the zenith, north and

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south poles; let O be the middle point of N S. Then O C, at right angles to N S, contains the centres of all the hour-circles, or meridians, through N and S. Let h be the hour-angle of the circle whose centre is C, then, by the angle property of the projection

$$\angle \mathrm{OCN} = \angle \mathrm{ONT} = h$$

and

$$\mathrm{OC} = \mathrm{NO} \,.\, \cot h$$

or

$$\mathrm{OC} = 2r \,.\, \operatorname{cosec.} c \,.\, \cot h$$

$$= \text{distance of centre from prime meridian.}$$

It is sufficient geometrically to fix the centre C, since the required circle must pass through N. But the radius

$$\mathrm{NC} = \mathrm{NO} \,.\, \operatorname{cosec.} h = 2r \,.\, \operatorname{cosec.} c \,.\, \operatorname{cosec.} h.$$

It is convenient when the circle becomes very great to determine the position of the point D. Now,

$$\mathrm{DO} = 2r \,.\, \operatorname{cosec.} c\,(\operatorname{cosec.} h - \cot h) = 2r \,.\, \operatorname{cosec.} c \,.\, \tan \frac{h}{2}$$

To draw such a circle practically it is best to make a template of card, with its edge forming part of a circle of the required radius;* if, then, the edge of the template is made to rest against the points N and D, this template may be used as a ruler to draw the circle on the diagram. In the diagram constructed, this was done when O C began to exceed five feet. The length of O C in the case of the last hour-circle, 5° or 20^{m} distant from the meridian, would be more than 19 feet. The last four circles were practically filled in by calculating the ordinates corresponding to nine equi-distant points, at intervals of $2\frac{1}{2}$ inches along A B, and these points were joined by straight lines. These polygons are not easily distinguishable from their circumscribing circles.

The results of the calculations for the diagram constructed are these,—

Latitude $52\frac{1}{2}°$ (Birmingham or Berlin),

A B = 25 inches, N Z = 4·243 inches, S Z = 36·824 inches.

Hence,

N S = 41·067 inches, N O = 20·5335 inches, Z O = 16·290 inches.

Radii of same-zenith-distance circles.

	Radius.	z	Radius.	z	Radius.
5	0·545	35	3·941	65	7·963
10	1·094	40	4·550	70	8·753
15	1·646	45	5·178	75	9·592
20	2·204	50	5·829	80	10·489
25	2·771	55	6·507	85	11·454
30	3·349	60	7·217	90	12·500

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* Suggested by Prof. Cayley.

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Radius, and distance (north of zenith) of the centre of each same-polar-distance circle.

Δ	Centre-Dist.	Radius.	Δ	Centre-Dist.	Radius.
5	4·252	0·609	70	6·702	10·345
10	4·279	1·221	75	7·232	11·476
15	4·325	1·839	80	7·869	12·730
20	4·391	2·467	85	8·642	14·142
25	4·477	3·108	90	9·592	15·756
30	4·592	3·760	95	10·775	17·633
35	4·719	4·446	100	12·279	19·864
40	4·880	5·152	105	14·236	22·588
45	5·065	5·897	110	16·860	26·025
50	5·299	6·667	115	20·525	30·558
55	5·567	7·491	120	25·877	36·902
60	5·883	8·370	125	34·624	46·590
65	6·258	9·342			

Perpendicular distances from the meridian of the centres of successive hour-circles.

h h m	Distance.	*h* h m	Distance.	*h* h m	Distance.
0 20	234·699	2 20	29·325	4 20	9·575
0 40	116·451	2 40	24·471	4 40	7·371
1 0	76·632	3 0	20·534	5 0	5·502
1 20	56·415	3 20	17·230	5 20	3·621
1 40	44·034	3 40	14·378	5 40	1·796
2 0	35·565	4 0	11·855	6 0	0·000

To test the accuracy of the diagram actually constructed, it may be noticed that four circles seem to meet in one point where

$$h = 60^\circ, \qquad \Delta = 75^\circ, \qquad z = 60^\circ, \qquad A = 75^\circ.$$

Calculation proves the true corresponding values to be

$$z = 60^\circ\ 2'{\cdot}6 \qquad A = 74^\circ\ 54'{\cdot}6.$$

Again, four circles seem to meet near a point where

$$h = 70^\circ, \qquad \Delta = 45^\circ, \qquad z = 45^\circ, \qquad A = 110^\circ.$$

Calculation gives,

$$z = 44^\circ\ 54'{\cdot}6, \qquad A = 109^\circ\ 45'{\cdot}1.$$

Upon the diagram there are about 650 intersections of polar-distance circles with hour-circles *east* of the meridian. The practical observer might, perhaps, find it best to measure on his diagram, once for all, the corresponding zenith-distances and azi-

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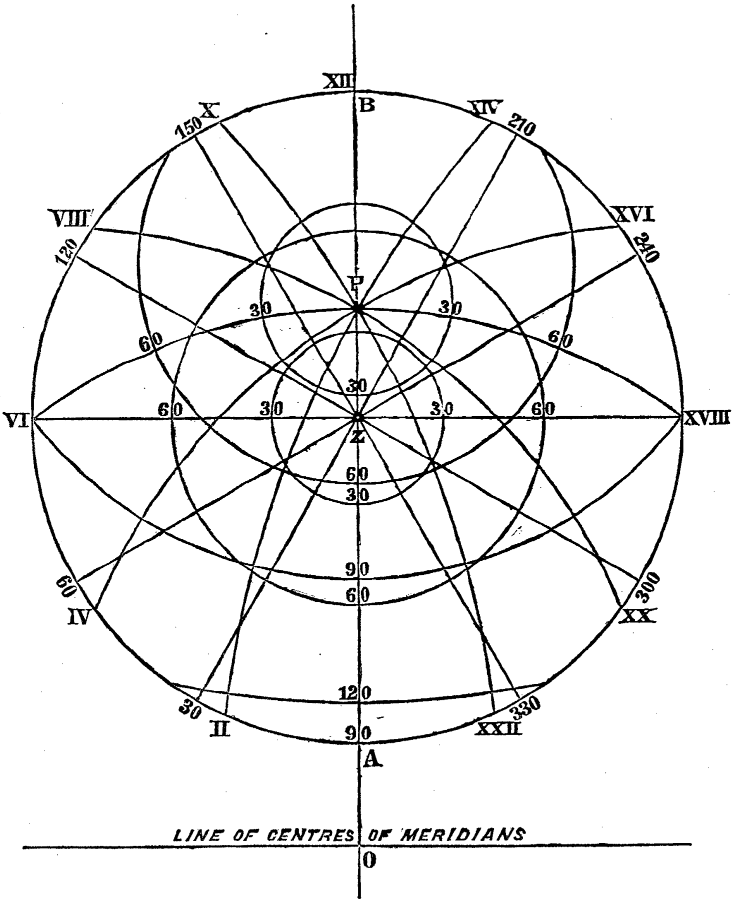
muth, and tabulate them for reference. The table would then supersede the diagram.

Stereographic projection for the conversion of polar-distance and hour-angle into zenith-distance and azimuth.



(Fig. 3). Lat. 52½°.

If it were considered advisable, diagrams such as the present could easily be constructed, for use with the diagonal altazimuths at each of the five English stations for the observation of the transit of *Venus* in 1874.

An abridgment of the diagram for latitude 52½° is shown in fig. 3.

St. John's College, Cambridge,
October, 1872.

On the Examination of the Photographs taken during the Total Solar Eclipse of December 11–12, 1871, *at Dodabetta.* By Lieut.-Col. J. F. Tennant, R.E., F.R.S.

In the text of my report on the eclipse, I have expressed my opinion that the relation between some of the prominences and